Title of Research: A Jaworskian Analysis of Four Senior Class Primary Teachers Endeavouring to Teach Mathematics from a Constructivist-Compatible Perspective.

Name of Researcher: Dr Joseph McCarthy

Research Idea:

The idea was to assist teachers in the implementation of a group-based problem solving approach to mathematics in $5^{th}/6^{th}$ classes over a one-year period.

Research Aims and Constructivism:

Research Aims

(a) The research aimed at moving teachers away from a textbook-centred approach to mathematics towards a problem-based approach. What are the implications of a problem-based approach?

(b) The research aimed at motivating teachers to ponder on the implications of a constructivist approach to mathematics as espoused in the 1999 curriculum. How does constructivist theory inform a problem-based approach?

(c) The research aimed at assisting teachers to move away from a teacher-led didactic approach to one which encourages pupils to develop their own understandings through problem solving activities. In what settings can pupils be best encouraged to develop such understandings?

To achieve these aims Jaworski's (1994) framework of the Teaching Triad was used as both an analytical and a developmental tool. The Triad helped to analyse the data and simultaneously keep the focus on developing teachers' practice. This involved looking at teachers' actions in the classroom under the three headings of management of learning (how

the classroom was organised), sensitivity to students (how pupils' learning was differentiated) and mathematical challenge (how pupils were cognitively engaged). Such analysis requires a bit more to be said on the theory of constructivism which underpins the primary curriculum.

Constructivism

Vygotsky's (1978) stance on learning is that it presupposes a specific social nature and a process by which children grow into the intellectual life of those around them. This view of learning falls within social constructivist theories of learning whereby students learn by actively making sense of new knowledge, deriving meaning from it and fitting it with their existing knowledge maps or schema. Although social constructivists contend that learning is a social activity and that learners construct their own meaning, a symbolic view of cognition still prevails and mind is still located 'in the head' (Cobb, 1999, p. 135). Therefore, learning and meaning are co-constructed but eventually this learning gets placed back within the individual. Moreover, summative or formative assessments (the latter favoured by social constructivists) are still measuring something that is the property of the individual. Learning is still the internalization of external knowledge, and what the student can do alone having learned through social interaction (Vygotsky, 1978). The 1999 curriculum does not specifically mention social constructivism, preferring to use the generic term 'constructivism', but it seems to advocate the social constructivist approach through the promotion of groupwork.

Not every author in mathematics education is an ardent fan of the application of constructivist theory to teaching. Schoenfeld (2006) is quite pessimistic about the research prospects for constructivist teaching. Indeed, he captures the breadth of interpretations of constructivist teaching when he states that it can mean "anything from diagnostic and prescriptive instruction to pretty much anything goes" (Schonfeld 2006, p. 201). He elaborates by

remarking that constructivist teaching, despite its faddishness, was an oxymoron to begin with; and once an oxymoron, always an oxymoron. Presumably, Schoenfeld prefers to focus on constructivism as being limited to a theory of learning. He elucidates by describing what learning to become competent in a domain includes:

> The development of a knowledge base; the ability to employ domain-specific and general problem solving strategies; the development of productive metacognitive behaviours such as monitoring and self-regulation; the development of one's own identity, including membership in various communities of practice; the development of productive beliefs and dispositions; the ability to participate productively in the practices of the domain, including discourse practices.
>
> (Schoenfeld 2006, p. 201).

Comprehensive and all as this definition of learning is, it fails to answer one vital question: what is the teacher's role in all of this? For me, Schoenfeld has ignored the interdependence and interplay which occurs between teaching and learning. I prefer to take the view that one cannot describe learning without referring to the resultant implications for teaching. Otherwise, learning will be seen as the sole responsibility of the learner herself, even if appearing to work within a community of practice as outlined above. Such a view is somewhat isolationist. In some languages the word for teaching and learning is the same. In Dutch, for instance, the distinction between teaching and learning is made only by the use of a different preposition. The verb is exactly the same. *Leren aan* means teaching; *leren van* means learning. Fosnot and Dolk (2005) remark that when teaching and learning are so closely related, they should be integrated in learning/teaching frameworks. As a result, teaching should be seen as closely related to learning, not only in thought and language but also in action. "If learning doesn't happen, there has been no teaching. The actions of teaching and learning are inseparable" (Fosnot and Dolk 2005, p. 175).

Another justification for investigating constructivism from a teaching point of view comes from the guidance given to teachers in the revised mathematics guidelines published in 1999. The guidelines recommend the use of scaffolding as one form of instruction. This is where the teacher modifies the amount of support according to the needs of the child. Initially, the teacher may expose the child to various possible methods of approaching a problem. Thereafter, "the teacher breaks down the task and makes the task manageable for the individual child, thus supporting the development of the child's own problem-solving skills" (NCCA 1999, p. 4). The guidelines also suggest that it is through social interaction that children can begin to appreciate the viewpoints of other people. The guidelines explicitly state that "sociocultural theory sees cognitive development as a product of social interaction between partners who solve problems together" (NCCA 1999, p. 4). Therefore, the role of significant others, such as peers, parents and teachers, in no particular hierarchy, in influencing children's learning is acknowledged. The terms used in the guidelines such as 'scaffolding', 'social interaction' and 'solving problems together' imply that the underlying philosophy of the mathematics curriculum is social constructivism, although the term itself is never used. It makes sense to tease out and explore the implications of such a philosophy for classroom teachers. Otherwise, terms like 'scaffolding', 'social interaction' and 'solving problems together' will remain as slogans but with no real, practical meaning for teachers.

My rationale, therefore, in exploring constructivism is to shed some light on a theory, which has been designed for learning but has not been given sufficient attention with regards to its implications for teaching. I view teaching and learning as being two sides of the one coin. "You can't have one without the other," says the old Sammy Cahn song. The Dutch seem to take the same view. This research hopes to explore constructivism and its implications but from the teacher's perspective. If constructivism is indeed a theory of learning it seems

reasonable to explore the pedagogical knowledge, which teachers need to learn, if they are to adopt constructivist-compatible pedagogies in the classroom.

7. Methodology Used Including Ethical Considerations

Data collection methods and data analyses

I now outline the data collection methods and data analyses which best suited this design research intervention. Firstly, I need to outline the framing of the research, as well as the timescale and participatory issues.

Framing the research: timescale and participants

The research was carried out during the 2010/2011 school year. Pseudonyms were used for all participants. Two were $5^{th}/6^{th}$ class teacher participants named Aoife and Claire from a large, middle class suburban primary school and two were $5^{th}/6^{th}$ class participants named Lisa and Anita from a medium-sized, disadvantaged suburban primary school. All teachers had between four and eight years teaching experience. Their participation was subject to all teacher and pupil participants signing an informed consent agreement.

The variety of data collection methods and data analyses

The following data collection methods and analyses were employed:

1. Initial semi-structured interviews based on questionnaires

I interviewed each teacher at the start of the project to ascertain their views on constructivism based on pages I had asked them to read from the Mathematics Curriculum (p.5) and Teacher Guidelines (p. 3-4) as I could not take it for granted that the teacher participants had any previous knowledge of constructivist approaches. This was done using a pre-lesson interview (PLI) questionnaire (see Appendix 1) which I had piloted with a teacher called Jim

(pseudonym). The questionnaire was used as a device to encourage teachers to talk about their views on constructivism. The interview was semi-structured with questions open enough to allow for individual interpretation. To this end, I am grateful to Jim, as I discovered that my initial questions were too closed and did not give adequate scope for a participant to air his views fully. As a result, I included more questions of the 'to what extent?' and 'what do you think?' variety. Cohen, Manion and Morrison (2009) state that open-ended responses often show up the gems of information that might otherwise be lost through closed questioning. From an ethical viewpoint, they also comment that such responses put the responsibility for and ownership of the data much more firmly into respondents' hands. Jim did not later engage with the project as one of my teacher participants. The actual four participating teachers were also interviewed as a group at the end of the project in June 2011 using a similar, but not identical, questionnaire. This was called the exit interview (EI) questionnaire (see Appendix 2). This questionnaire was designed to evaluate if teachers' views of constructivist approaches had changed throughout the project; hence the similarity of questions to those in the pre-lesson interview. A third and final questionnaire (see Appendix 3) was applied in May 2012, almost a year after the project had ended, to ascertain if the project had had a long term impact (LTI) on the teacher participants' practice.

Oppenheim (1992) states that semi-structured interviews are designed to develop ideas and research hypotheses rather than to gather facts and statistics. This is particularly relevant when one is researching a topic as broad as constructivism. Cohen et al. (2000) comment that interviews enable participants to discuss their interpretations of the world in which they live and to express how they regard situations from their own point of view. The danger is that an interviewer can intervene with his own views and cause bias to emerge. I attempted to remain as impartial as possible during the interview process. Transcribing and analysing such

interviews is also extremely time-consuming. As interviews can be subjective in terms of what is deemed relevant by the interviewer it is important that other sources of data are collected also.

In terms of analysis I colour coded participants' responses to questionnaires to ascertain if their views on constructivism had changed during the course of the project. These views were then categorised under the three headings of Jaworski's Teaching Triad: Management of Learning, Sensitivity to Students and Mathematical Challenge. It has to be stated that these headings overlap. An instance of such overlap from the questionnaires came when I asked participants for their views on scaffolding. I categorised such views under the heading Sensitivity to Students. It could be argued that scaffolding fits under either of the other two elements of Jaworski's Triad. Therefore, it can be seen that I had to categorise the data based on personal interpretation of which element suited best. In the pre-lesson and exit interview questionnaires I had to apply Jaworski's Teaching Triad as the analytical tool. However, the long term impact questionnaire from May 2012 is blatantly Jaworskian in format as I had the benefit of hindsight and knew that the Teaching Triad categories were probably the most useful way of analysing the data.

2. Open post-lesson audio-taped interviews with videotaping of lessons as background.

As stated earlier, I observed each of four teachers for a total of four lessons each. Each lesson observed was videotaped using a Cisco Systems Flipshare Mino HD recording device. The advantage in using this device was that it could record sixty minutes of footage (adequate for most lessons) and had an attachment for loading such footage onto a computer. Immediately after the observed lesson the teacher was interviewed for five to ten minutes. The only set question for the interview was 'how do you believe the lesson went for you?' The purpose of the question was to initiate the process of tracking the teacher participants' emerging views

on constructivist approaches and to provide advice where needed. Here I took the counsel of Kvale and Brinkmann (2009, p. 167) who recommend that an interviewer should be open-minded so that (s)he "hears which aspects of the interview topic are important to the interviewee, listens with an evenly hovering attention, and is open to new aspects that can be introduced by the interviewee and follows them up". In terms of analysis these interviews were important to the design of the project as they determined the type of design intervention required. The analysis therefore consisted of identifying the topics of concern to the teacher whose lesson had just been observed and providing appropriate advice on how to proceed. For instance, if a teacher's concern was that the mathematical activity chosen was too closed I suggested a more open-ended task. Most of this advice was instantaneous but I had the benefit of reviewing the videotape later that day to see if any further advice was needed. This advice could then be communicated by phonecall or text message. The oral interviews were recorded using an Olympus VN-3000PC digital voice recorder. These post-lesson interviews (POLI) were an invaluable source of data because one of the participants, Aoife, chose not to write written reflections on her lessons and, as a result, I was dependent on her oral testimony for data collection. After each lesson I loaded the videotape onto the teacher's laptop and they were encouraged to view it and include their insights in their post-lesson reflections written in their teacher journals.

3. Teachers' written reflections

Teachers were also encouraged to keep notes on aspects of lessons which had proved insightful for them in gaining an understanding of constructivism. I had hoped that this would be a useful way of collecting data when teachers were teaching off-camera. However, the teachers chose to use their journals solely to record their written reflections on the videotaped lessons. This would later indicate that they did not engage extensively with constructivist approaches once the camera was not present. The abbreviation used for the

reflections on the lessons is RL so, for instance, LRL1, refers to Lisa's reflection lesson 1. As regards analysis, the written reflections would later be categorised under Jaworski's Teaching Triad and form part of the synthesis of the research on teachers' evolving views of constructivist approaches. In real time the reflections helped the teachers and me to plan the format of the next lesson to be observed. These observational lessons were scheduled for times and dates convenient to both teacher participants and researcher.

4. Researcher's journal entries

I kept a journal and included comments in it for each lesson observed. In terms of analysis these comments helped me to co-design a constructivist path with each individual teacher participant. Taking notes on classroom observations is not without its dangers. Cohen, Manion and Morrison (2009) warn that the presence of the observer might bring about different behaviours. Indeed, interviews with the children later confirmed this to be the case with some of the participant teachers. Furthermore, the researcher is in danger of 'going native' or "becoming too close to the group to see it sufficiently dispassionately" (Cohen, Manion and Morrison, 2009). Therefore, the notes I took were used in conjunction with my viewings of the videotapes to enable me to reflect on both the practice and theory of constructivist approaches and to assist me in deciding which questions to raise and the type of dialogue to pursue with each teacher participant in the pre- and post-lesson interviews. As a result, my views on teachers' engagement with constructivist practices evolved lesson by lesson. I could be described as a participant observer of the lessons without being an interactive participant in them.

5. Pupils' work samples

I believed that when the teacher participants gave their views on constructivism during the project validity would be enhanced if such views could be supported by relevant work

samples from the pupil participants. These samples would help teacher participants to reflect on their practice and observe if pupil understanding was improving in line with the relevant changes in teacher work practices. In terms of analysis I chose the work samples based on the Vygotskian construct of the zo-ped to illustrate the mathematical challenge required of the pupils in the tasks undertaken. Mathematical challenge is one of the elements of Jaworski's Teaching Triad.

6. Focus group interviews

In line with a social constructivist view of knowledge construction I had hoped to hold several focus group interviews throughout the project in which all teacher participants could share their views on constructivism. However, the logistics of getting all teacher participants together at the same time after school hours proved very difficult. Therefore, I managed to hold only one focus group interview at the end of the project in June 2011. At the very least, this interview yielded data on the challenges faced by the teacher participants as they engaged with constructivist-compatible approaches to their work. It also helped me to theorise on how best to prepare teachers for such a pedagogical assault on a constructivist obstacle course. However, I was able to hold a second focus group interview, almost a year later, in May 2012 to ascertain the long term impact (if any) on teacher participants' practice. I took the 2011-2012 academic-year out of my studies due to my wife suffering a protracted illness. However, my supervisor, Dr. Paul Conway, suggested that a long term impact interview could be a useful way of getting back into my studies. I thought this was a good idea and set up the group interview. In referring to the data I used the abbreviation LTIQ to refer to the long term impact questionnaire and the long term impact interview transcript appears in the original thesis.

7. Pupil interviews and ethical considerations

Seeing as constructivism is a theory of learning I believed it was imperative to obtain some of the pupil participants' views of the project. This was done by interviewing groups of three pupil participants, of varying abilities, from each of the four classrooms. The interview was open and in it I just asked the pupils for their impressions of the project and by extension their views of teacher behaviour during the project. Here I took Kvale and Brinkman's (2009) advice to avoid "long and complex questions and posing more than one question at a time" (p. 146). I did not propose to interview children individually, as this could have been intimidating for them. Eder and Fingerson (2002) draw attention to the power imbalance which exists between teacher and pupil and stress the need for the interviewer to avoid being associated with the classroom teacher. Therefore, I interviewed the children away from the classroom but in a room familiar to them, such as the learning support room or the computer room. Cohen, Manion and Morrison (2009) state that group interviewing "encourages interaction between the group rather than simply a response to an adult's question" (p.374). They also surmise that it is less intimidating for children than individual interviews. This proved to be the case as the children smiled and appeared relaxed during the interviews. Perhaps, they were also glad to be avoiding class work. In giving their written consent for the project the children (and their parents) knew they could be asked to take part in a group interview but that their participation was entirely voluntary. In analysing the children's views of the project I again applied the Vygotskian construct of the zo-ped to ascertain if there had been challenge for the children in the tasks undertaken. It will be remembered that I also applied this lens in choosing children's work samples. As an aside, the children gave interesting insights into how teachers deviated from their normal behaviour during the project and these are reported in the original thesis. In Table 1 below I offer a summary of the abbreviations I used in the research process and in Table 2 I give a chronology of the data

collection. Please note that abbreviations may be combined also; so for instance, CPOLI4 would mean Claire's post-lesson interview for her fourth lesson.

Table 1: A list of data collection abbreviations

Abbreviation	**Meaning**
A	Anita
AO	Aoife
C	Claire
EI	Exit Interview

GI	Group Interview
L	Lisa
LTII	Long Term Impact Interview
PLI	Pre-lesson Interview
POLI	Post-lesson Interview
RL	Reflection on Lesson

Table 2: Chronological data collection timetable

Chronological order of data collection	
Initial semi-structured interviews	School year 2010/2011
Videotaping of lessons with post-lesson discussion	School year 2010/2011

Teachers compile written reflections	School year 2010/2011
Researcher compiles journal entries	School year 2010/2011
Collection of pupils' work samples	School year 2010/2011
First round of focus group interviews	June 2011
Interviews with pupil-groups of three	June 2011
Second round of focus group interviews	May 2012

Overview of data analysis procedures

I have already referred to the variety of data sources. In approaching the study from a constructivist perspective, I was aware that the researcher creates the categories and concepts through interaction with the field-grounded theory approach, which attempts to build categories and concepts emerging from the data (Glaser & Strauss, 1967; Strauss & Corbin, 1990). Charmaz (2000) recommended adopting more of a constructivist approach to grounded theory that recognises that the categories and concepts are not inherent in the data, waiting for the researcher to discover them. Instead, it is the researcher who creates the categories and concepts as the result of interaction with the field and the questions that are asked. I looked at what was built through a critical constructivist lens, so that current theory on constructivist-compatible approaches could be affirmed, denied or reformulated in some way. It could be said that the process of induction used to build the data had to be subjected to a process of deduction to see if the data elaborated on issues pertaining to constructivist theory. Such issues included pupils' engagement, as individuals or in groups, at an intellectually challenging level, as manifestation of working at the zone of proximal development. Also included would be the issue of how the teacher scaffolded the learning to

reach such a high level of engagement. Computer-assisted software was not used in the analysis of data gathered. Instead, I used colour-coding to cross-reference data collected from classroom observations with that collected from teachers' journals and interviews. The categories of Jaworski's Teaching Triad were used as the dominant overarching analytical tool. My rationale was that I believed that the categories of the Teaching Triad were broad enough to encapsulate any data generated and any software searches for key words or phrases would not adequately convey the complexity of the classroom interactions. As mentioned above, the data emerging from the use of the Teaching Triad as an analytical tool still had to be compared to the literature on constructivist theory for affirmation or denial. A summary of the data analysis process showing the emerging themes is given below in Figure 1.

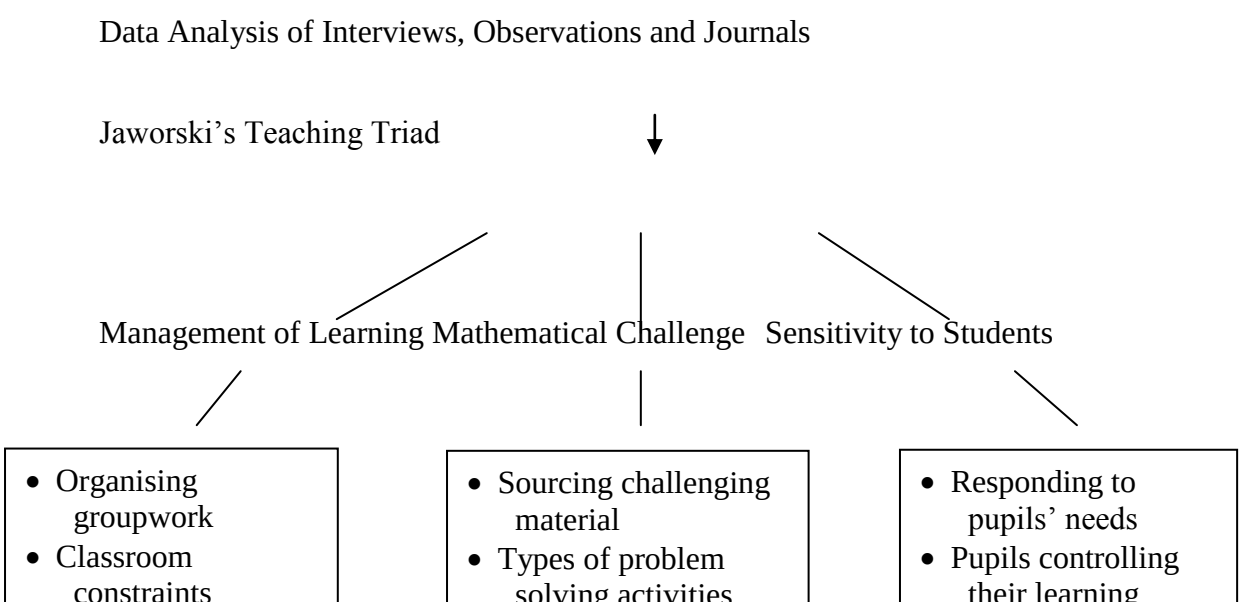

Figure 1: Data analysis of interviews, observations and journals

8. Summary of Background Reading

A Focus on Current Questions

Brophy (2006) states that "social constructivist educators usually have much more to say about learning than about teaching" (p. 530). It is this gap in the literature that interests me. It is all very well to ponder on whether the curriculum comes from a constructivist or a social constructivist epistemology, but the real issue affecting teachers is how to teach in a constructivist-compatible approach whatever the epistemological variations. The strength of the literature to date is that there has been a strong focus on how children learn. The weakness is that there has been little focus on how teachers should teach when they wish to adhere to a constructivist approach to teaching. Brophy (2006) 'hits the nail on the head' when he writes that "it is unrealistic to educate teachers to implement social constructivist principles without systematizing them into operational models of teaching" (p. 530). When I conducted a "deep see" trawl of the literature I was intrigued to find several authors who have tried to address the issue of constructivist-compatible *teaching* as opposed to *learning*. I now wish to give a brief summary of the work of such pragmatic authors.

Brophy (2006) outlines Graham Nuthall's seven principles for effective implementation of social constructivist teaching. What is interesting about his views is that they include aspects such as "ensure frequent repetition" and "repeat critical content" (p. 533). The point here is that Nuthall does not see teaching as transmission as being mutually exclusive to a social constructivist approach.

Gagnon Jr. and Collay (2001) spoke of a constructivist learning design (CLD) "composed of six basic parts flowing back and forth into one another in the actual operation of classroom learning: situation, groupings, bridge, questions, exhibit and reflections" (p.xi). The authors draw attention to the surface activation of students' prior knowledge before introducing them to new subject matter. They also stress the importance of the teacher providing questions, which instigate, inspire and integrate students' thinking and sharing of information. Teachers' questions usually fall into open or closed categories. This can be a useful indicator of whether pupils are being allowed to construct their own knowledge or being funnelled into the teacher's set knowledge. Furthermore, Gagnon Jr. and Collay (2001) highlight the use of groupwork more than Nuthall does (Brophy 2006).

A third author who grappled with what a constructivist approach to teaching might entail is Jaworski. Her work was to have a profound effect on how I analysed my own classroom observations. In the classroom study, which formed the bulk of her PhD thesis, she found it useful to analyse teachers' modus operandi in terms of what she called "the Teaching Triad" (1996, p. 107). The three domains of the Teaching Triad were management of learning, sensitivity to students and mathematical challenge:

> Management of learning is manifested in a set of teaching strategies and beliefs about teaching which influence the prevailing classroom atmosphere and the way in which lessons are conducted. Sensitivity to students in inherent in the teacher-student relationship and the teacher's knowledge of individual students and influences the way in which the teacher

> interacts with, and challenges, students. Mathematical challenge arises from the teacher's own epistemological standpoint and the way in which she offers mathematics to her students depending on their individual needs and levels of progress.
>
> (Jaworski, 1996, p. 108)

Jaworski has advanced my thinking in that she gives some guidelines as to what to look for in constructivist classrooms. It is useful to link her Teaching Triad categories with the advice given by Gagnon Jr. and Collay (2001). The following questions come to mind as regards observation of teachers:

1. Does the teacher *manage learning* in such a way that the classroom atmosphere is conducive to optimum learning? In this regard Gagnon Jr. and Collay (2001) would suggest the use of groupwork.

2. Is the teacher *sensitive* to pupils' needs? Gagnon and Collay (2001) would suggest helping the pupils to build mental bridges to enable them link their prior knowledge with the new subject matter.

3. Is there *challenge* for the pupils mentally in the work undertaken? It is here that Gagnon and Collay (2001) suggest that questions need to be inspiring for pupils and helpful to them in integrating their thinking.

Constructivism is a theory of learning, not a theory of teaching, and many educators who attempt to use reform pedagogical strategies confuse discovery learning and "hands-on" approaches with constructivism. For instance, children may be observed engaging with a mathematical problem using manipulatives. However, this does not necessarily mean that they are operating at their zone of proximal development. Holt-Reynolds (2000) gives another illustration of this misconception when she describes a prospective teacher named Taylor. In her classroom Taylor used active learning methods such as encouraging pupils to offer their opinions during English literature class. However, such opinions were not

challenged by the teacher or other students. Taylor made the mistake of equating participation with learning. She needed to "see constructivist pedagogies as techniques for teaching, not merely as strategies for activating kids" (Holt-Reynolds (2000, p. 30). In other words, activation is an essential aspect of extrinsic motivation but it does not follow that activation will ensure pupils are cognitively challenged. Such challenge is at the heart of reform mathematics which seeks to move pupils beyond the mundane problems inherent in school mathematics textbooks. The primary curriculum and Project Maths programmes aspire to a 'minds-on' and not just a 'hands-on' approach to mathematics. Gagnon and Collay (2001) use the phrase 'mental bridges' to refer to the linking of prior knowledge with new subject matter. However, this needs to be an ongoing process so that pupils are constantly reinventing and reinterpreting their knowledge. Certainly, the use of sociocultural tools, like calculators, computers and the internet have a role to play in helping pupils to expand their 'mathematical horizons', to quote Ball's (1993) phrase. It can be seen that I am trying to weave several authors' writings into a constructivist framework which would enable me look at current practitioners' classroom teaching in terms of its relevance or otherwise to constructivist theory. The contribution of several authors has been considered. These authors include Brophy (2006), Gagnon Jr. and Collay (2001) and Jaworski (1996b).

9. Overview of research findings and recommendations

Recommendations for classroom practice

These implications for practice are written in the form of advice for an audience of teachers interested in adopting a constructivist-compatible approach in their mathematics classrooms. They are not written in any particular hierarchy of merit. One of the limitations of the

research is that only four classrooms were involved and therefore it is difficult to generalise the findings to cover a wide variety of contexts. Nevertheless, I believe teachers who wish to move the teaching of mathematics beyond the routine coverage of a textbook will find the following advice valuable:

1) **Pupils' attitudes and cognitive challenge:** When teachers adopt an investigative group-centred approach in their mathematics classrooms, pupils' attitudes towards mathematics seem to improve. Pupils describe the resultant mathematics experience as being more fun. One proviso I would include is that the teacher needs to be vigilant that pupils are working at a challenging level and not just 'coasting' in their groups, waiting for the higher-achieving pupils to come up with all the correct procedures. Stigler and Hiebert (1999) quote an unnamed mathematics education professor in their video study of 81 U.S. classrooms. The professor states, "In U.S. lessons, there are the students and there is the teacher. I have trouble finding the mathematics; I just see interactions between students and teachers" (p. 26). The point is that the interactions are not enough; there must also be cognitive challenge for the pupils. This thesis has stressed the need for cognitive challenge in activities presented to pupils; going so far as to use Jaworski's categorisation of mathematical challenge as one of the developmental and analytical tools throughout the study.

2) **Organising group work:** The organisation of group work was a major issue for the participant teachers. From a constructivist viewpoint a teacher hopes that splitting a class into groups will ensure that every group and every individual within a group gains an important amount of sustained attention of the kind that can produce the 'higher-order cognitive interactions' that Galton (1980) and Mortimore (1988) regarded as essential for purposeful learning. In general, the teacher participants thought that grouping suited the higher-achieving pupils more than the lower achieving ones. For instance, Aoife thought there were times when placing the higher achievers together meant that they could 'expand more on an

activity'. However, Lisa thought that lower achieving pupils benefited more in groups as they gained assistance from higher achieving ones. The point I wish to make here is that it is difficult to generalise as to which groupings of pupils work best; seeing that affective as well as cognitive factors have to be considered in varying contexts. Teachers need to experiment with different groupings of pupils and adapt accordingly when pupils are not working at their optimum level. I believe there is a great need for representative pupils to share what they have learnt in groups with other members of the class. In other words, I am advocating plenary sessions where knowledge is shared in line with an emergent perspective. The reader may recall Anita's strategy of 'harvesting and sowing' whereby the useful ideas from one group were gathered and disseminated to other groups by the teacher. She also dispersed ideas through the strategy called 'Move About' which allowed pupils to move from one group to another. These strategies counteract the traditional modus operandi of children completing individual, humdrum written tasks at their desks during lessons.

3) **Teaching as telling:** Teachers in Ireland work with the second largest class sizes in Europe. The pressure under which teachers put themselves to cover a set curriculum in such large classes was palpable and readily perceptible to this researcher. The DES study entitled An Evaluation of Curriculum Implementation in Primary Schools (2005) found that teacher-talk dominates as a methodology. Purists may argue that this is incompatible with a constructivist approach to learning. However, I have seen that there are times, even in investigative problem-based mathematics, that it is an expedient and efficient use of precious time to tell pupils information, which may help them in an investigation. The dilemma for the teacher remains *what* to tell and *when* to tell it.

4) **Teacher change:** Although I have to report that once the project ended, I do not believe the participant teachers adopted a constructivist-compatible approach to their work for the longer term, I believe that their knowledge of what such an approach entailed had increased.

From what the teachers stated in the long term interviews, their awareness of pupils' varying learning styles had been heightened and a lasting effect was that they had become more open to letting pupils derive and describe their own individual methods. This showed willingness on the part of the participant teachers to relinquish some control and allow pupils construct their own problem solving meanings. The teachers could be described as 'part time' advocates of a constructivist approach. In similar findings to O'Shea and Leavy (2013) I found that the teacher participants placed a significant focus on computation and recall of basic mathematical facts. Such a focus remains deeply embedded in the teaching practices of teachers. Teacher change is a notoriously slow process. Ross et al. (2002) are graphic when they comment that progress towards "implementing reform ideals will be incremental, with advances occurring on a broken front with many backward steps" (p. 131). I agree with O'Shea and Leavy (2013) who comment, from their research in Irish primary classrooms, that although teachers were inspired by learning from a constructivist perspective, it was evident that methodologies that reflect constructivist principles would not usurp the traditional methodologies used by such teachers. There are many reasons for this finding. In this research Claire blamed the heavy workload in 5^{th} class and the unavailability of suitable problem solving activities. Aoife stated that she had to teach formulas and problem solving methods before she could consider taking on any other type of mathematical activities. On a similar note, Lisa thought that teaching problem solving methods to enable pupils to cope with standardised tests like the Sigma-T dominated her pedagogy. Anita thought that there were too many subjects to be covered in the curriculum and this militated against her finding the time to scan the internet to find suitable problem solving activities. It seems that teachers' perceptions of what mathematics entails and the time available to teach the subject, according to those perceptions, acts as a barrier to their adoption of constructivist-compatible approaches.

4) **Choosing appropriate activities:** One of the difficulties the teachers experienced during the project was the sourcing of appropriate material for the problem solving activities. They thought that it would be extremely difficult to adopt a constructivist-compatible approach if they had to spend time searching the internet and looking up other textbooks to find suitable activities. They wanted readymade material to hand. Such views were also voiced by teachers at a problem solving workshop I attended during the Irish National Teachers' Organisation Education Conference held on 15/16 November 2013. The theme of the conference was Numeracy in the Primary School and the workshop was facilitated by Dr. John O'Shea, a lecturer in mathematics education and teaching methodology at Mary Immaculate College, Limerick and whose work I have quoted in this thesis. I believe those teachers' views reflect an over-reliance on standard textbooks and a lack of value on the merits of engaging in open-ended problem solving activity. Such a reaction is understandable from teachers who are already very busy teaching routine algorithms and standard textbook problem solving methods. The teacher participants believed they were under pressure to cover such prescribed content and report on the amount of material covered in their monthly reports (cuntaisí míosúla). There is very little emphasis in the curriculum on open-ended problem solving. If such problem solving was valued more and emphasised more in curriculum documents and in-service courses, I believe it would get more attention from teachers. Teachers could then be expected to draw on their professionalism in sourcing additional material. If investigative mathematics is valued by teachers it will be covered by teachers. Both the Department of Education and Skills and the National Council for Curriculum and Assessment have a role to play in the promotion of investigative mathematics. From the American perspective Schoenfeld (2001) called for greater alignment of curriculum objectives with reform standards. As Alexander (1995, p.306) remarks,

"Curriculum balance, then, is a product of decisions taken across the system as a whole, not merely within the school and classroom. It is a matter for policy-makers as well as teachers."

5) **The importance of linkage and connectivity:** A possible means of achieving a balance between covering a set curriculum and engaging in investigative mathematics is for teachers to use the methodology of linkage more often. In that way teachers can link a problem solving activity with prescribed curricular objectives and ease their conscience somewhat when it comes to writing up monthly reports. In this thesis, I elaborated on the issue of linkage and broadened it into a concept called Connectivity which would expand Jaworski's Teaching Triad into a quadriad. For Connectivity to work a teacher must not only link investigative mathematics with the prescribed curriculum, as in traditional linkage, but must also link it with pupils' real world experiences of mathematical problem solving and with mathematical concepts emanating from other curricular subjects (integration). Such connection-making can be pre-planned but more of it should occur in the throes of a lesson when, for instance, a teacher spots a suitable opportunity to pursue a pupil's query.

Recommendations for policy on continuing professional development

O'Shea (2009) stated that the primary curriculum reflects the principles of the emergent perspective on constructivism, but that from a reader's perspective little background is offered to place its centrality to the curriculum in focus. Moreover, he argues that the presentation of the curriculum's content in clearly defined units places significant restrictions on teachers engaging students in learning from an emergent perspective. I would agree with O'Shea on both points. The issue then becomes a question of what can be done to improve matters. I have highlighted above how traditional linkage needs to be broadened into the concept of Connectivity, whereby teachers constantly seek out material which encourages pupils to make connections between mathematics and their own lives. How best to provide

in-service to teachers on constructivist-compatible approaches also becomes relevant. When I was a tutor on the Primary Curriculum Support Programme during 2001/2002 teachers received two days in-service on the revised mathematics programme. As this in-service was delivered to large groups of teachers in hotel rooms, it was not compatible with showing teachers how to divide a class into groups and engage the pupils in a problem solving activity. No wonder then that McCoy et al found in the Growing Up in Ireland Study (2012, p. 35) that "teachers of large classes are more likely to take more traditional approaches, perhaps reflecting greater logistical constraints and space constraints". O'Shea (2009) comments that successful in-service needs to be classroom-based with particular emphasis placed on prolonged periods of classroom support, which is consistent with current literature (Loucks-Horsely, Hewson, Love and Stiles, 1998). Ross et al. also (2002) state that the most powerful method for increasing implementation of reform mathematics is in-service. They add that it is essential to provide ongoing professional development, particularly focused on providing teachers with examples of constructivist teaching (Bitter & Hatfield, 1994) and explicitly addressing their beliefs about mathematics as a teachable subject (Grant, Peterson, & Shojgreen-Downer, 1996). O' Shea (2009) quotes Snyder, Lippincott and Bower (1998) who suggest that the most effective method employed in the professional development of beginning teachers is a practice-oriented model; where participants devise plans, implement them and reflect upon what happens. There is a strong emphasis in the Irish system on newly qualified teachers providing copious quantities of written notes in order to be certified as probated when a Department of Education and Skills (D.E.S.) inspector visits. Thankfully, D.E.S. Circular 39/12 heralded a more reflective approach with school staffs encouraged to identify areas in need of development and to decide on actions that should be taken to bring about improvements in those areas as part of their three-year literacy and numeracy plans. This means that schools can identify areas like problem solving in mathematics as being in

need of attention and give them full priority. Under the Haddington Road pay agreement (2012) teachers have to work an extra hour after school once every week. This means that teachers could prioritise the teaching of investigative problem solving, if the will to do so is present. With a shortage of support personnel on the ground from the main in-service provider, the Professional Development Support Service (P.D.S.T.), schools are forced to bring in outside expertise to help. Indeed, I have been personally asked to give in-service to several staffs in the area of mathematics problem solving. Hopefully, school staffs will use such in-service to gain insights on how to move away from a textbook-led approach towards a more investigative stance to mathematics problem solving. Such in-service is required, not only at primary level, but at second level also, where teachers need to explore investigative mathematics as envisaged in the Project Maths curriculum, as referred to in chapter three. The Project Maths curriculum was designed to follow on from the primary mathematics curriculum. The Dutch Realistic Maths Education (RME) movement has merit as a model for the teaching of investigative problem solving. That is not to say that such a change in teaching methods can occur quickly. Indeed, Stigler and Hiebert (2009, p. 87) state that cultural activities like teaching "evolve over long periods of time in ways that are consistent with the stable web of beliefs and assumptions that are part of the culture". Delaney (2012) adds that because teaching is a cultural activity, it is difficult to change through teacher education or professional development for teachers. He proffers that teaching is an activity that is absorbed from culture through family conversations over meals, through watching television and listening to radio, and of course from spending 13/14 years as a pupil in various classrooms observing teachers teach. I would add that, unlike other professions, everyone feels qualified to give an opinion on teaching as everyone has been through the education system. Delaney (2012) suggests, and I agree, that the way forward is for teachers to make small changes to their practice over time. For instance, he recommends the

development of new habits in mathematics classes; such as asking children to explain how they got their answers, or replacing textbook problems with open-ended problems from a site such as NRICH (or as participant teacher Lisa suggested; www.figurethis.org), or start referring to children as 'low achieving' at maths rather than 'weak' and 'high achieving' rather than 'strong'. His suggestions regarding the labelling of pupils became apparent in this research also. This research certainly heightened the teacher participants' awareness of allowing pupils to create various solution methods to problems and share them with their peers. Delaney (2012) comments that without changing our habits, we won't change a cultural activity like teaching. In this thesis I mentioned an inspirational programme for teachers called the Educational Leaders in Mathematics (ELM) Project as reported by Schifter and Fosnot (1993), which is a two-level programme. At the initial level, teachers who attend a two-week introductory summer institute receive weekly clinical supervision during the following academic year. Many of these teachers then proceed to an advanced level comprising a second institute and an apprenticeship programme in which they learn to conduct workshops for their colleagues. Such a system is possible in Ireland also. Primary teachers already receive an incentive of four Extra Personal Vacation (EPV) days if they partake of two weeks summer in-service. If the summer course could be followed by a year-long course in mathematics education, with school-based assignments leading to a diploma from a recognised university, then I believe teachers might be motivated to experiment with constructivist-compatible approaches. As I write this conclusion the INTO has advertised a summer in-service course entitled "Maths Problem solving: Process, Not Product". This type of course echoes the sentiment of this thesis and is to be welcomed. Another movement to be welcomed is Assessment for Learning (AfL) as outlined on the NCCA website. This movement promotes the integration of learning with assessment in a formative way, while still allowing for summative assessment at intervals. This type of assessment is in line with a

constructivist approach which sees learning and assessment intertwined. Ross et al. (2002) use the word 'integrated' to describe such assessment and see it as being "in contrast with end-of-week and unit tests of near transfer that characterise assessment in traditional programmes" (p. 125).

10. How the research has contributed to my professional development

This doctoral research has contributed greatly to my professional development in that I now know a lot more about the theory of constructivism and feel qualified to speak about its resultant implications for school policy and classroom practice.

11. How this research will benefit the teaching profession and wider community

The theory of constructivism underpins the 1999 mathematics curriculum. It has been criticised as being a vague theory by Schoenfeld (2006). This research elucidates the theory somewhat and as a result it benefits teachers who wish to adopt a constructivist-compatible approach to their classroom practice. Parents can also benefit from the strategies outlined in helping their children with problem solving activities at home. The wider research community will benefit from this research as it extends and expands on similar research on problem solving conducted by O'Shea (2009).

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Appendix 1:

Pre-Lesson Interview Questionnaire

Interviewee:_____ Date: _____ Time: _____

1. If you have read p. 3-4 of the Mathematics Teacher Guidelines and p.5 of the Mathematics Curriculum can you comment on any aspects which struck you in particular?

_____

_____

_____

_____

Can you give examples of any implications for the teacher in adopting a constructivist approach?

_____

_____

_____

_____

2. In your opinion how does constructivism affect what pupils do in classrooms?

_____

_____

_____

_____

3. “Work on open-ended problems, where the emphasis is placed on using skills and discussion rather than seeking a unique solution, is recommended” (Teacher Guidelines, p.4). What do you think of this statement?

_____

_____

_____

_____

4. Can you give examples of what you think are the main impediments to teachers adopting a constructivist approach in Irish primary mathematics classrooms?

_____

_____

_____

_____

5. Words like “scaffolding” appear in the Teacher Guidelines. Can you give an example of what this means to you?

_____

_____

_____

_____

6. From where has your own knowledge of constructivism come?

_____

_____

_____

_____

7. To what extent would you describe your current practice as constructivist? Please give examples.

_____

_____

_____

_____

8. Would you like to add any more comments on how you believe constructivism influences classroom practice?

_____

_____

_____

_____

Appendix 2:

Exit Interview Questionnaire

Interviewee:_____ Date:_____ Time:_____

1. Are there any aspects of the project which struck you in particular?

_____

_____

_____

_____

2. What do you now think are the implications for you in adopting a constructivist approach?

_____

_____

_____

_____

3. What do you now think are the implications for the pupils if the teacher adopts a constructivist approach?

_____

_____

_____

_____

What do you now think of working on open-ended problems?

_____

_____

_____

_____

4. Have the impediments to adopting a constructivist approach changed for you in any way?

_____

_____

_____

_____

5. What does the word “scaffolding” now imply for you?

_____

_____

_____

_____

6. Did you work on any open-ended problems off camera? If so, describe the experience.

_____

_____

_____

_____

Has the project influenced your practice in any way?

_____

_____

_____

7. Any other general comments on the project?

_____

_____

Thank you for your participation!

Appendix 3:

Long Term Impact Questionnaire

Name: _____ Class: _____ Date: _____

1. To what extent, if any, did the project influence the way you introduce mathematical challenge to pupils?

_____

_____

_____

_____

_____

2. To what extent, if any, did you become more sensitive to pupils' mathematical needs?

_____

_____

_____

_____

_____

3. To what extent, if any, did you change the way you manage your classroom?

_____

_____

_____

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4. To what extent, if any, did the project impact on your practice?

_____

_____

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_____

5. What advice would you give to another teacher attempting to adopt a constructivist approach to their work?

_____

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_____

6. Please write any other comment you may have on the project.

_____

_____

_____

_____

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_____

THANK YOU!